

# An imaginary temperature far away from a stationary spinning star

A. K. M. Masood-ul-Alam

Yau Mathematical Sciences Center

Tsinghua University, Beijing 100084, China

[abulm@math.tsinghua.edu.cn](mailto:abulm@math.tsinghua.edu.cn)

PIRT-2015, Bauman Moscow State Technical University, Moscow, Russia



# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdtd\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.



# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdtd\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.

Except for the occasional mentions of the universe, our spacetime is globally regular having space topology  $\mathbb{R}^3$  and having no ergoregion.



# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdtd\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.

Angular velocity of “the dragging of inertial frame” is  $\omega = -W/X$ .



# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdt d\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.

Angular velocity of “the dragging of inertial frame” is  $\omega = -W/X$ .

This means test particles with “zero angular momentum ” move along trajectories whose angular velocity relative to a stationary observer at infinity is  $\omega = d\phi/dt$ . Such an observer has 4-velocity  $\frac{\partial}{\partial t}$  in his proper frame.  $\omega$  is not the angular velocity of the fluid measured by the same observer.

# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdtd\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.

Angular velocity of “the dragging of inertial frame” is  $\omega = -W/X$ .

Energy-momentum tensor of perfect fluid is

$$T_{ab} = (\epsilon + p)u_a u_b + pg_{ab}, \text{ where fluid's (normalized) 4-velocity is}$$
$$u^a \frac{\partial}{\partial x^a} = \tau K^a \frac{\partial}{\partial x^a}.$$

# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdtd\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.

Angular velocity of “the dragging of inertial frame” is  $\omega = -W/X$ .

Energy-momentum tensor of perfect fluid is

$$T_{ab} = (\epsilon + p)u_a u_b + pg_{ab}, \text{ where fluid's (normalized) 4-velocity is}$$
$$u^a \frac{\partial}{\partial x^a} = \tau K^a \frac{\partial}{\partial x^a}.$$

It is known that in thermal equilibrium the fluid moves rigidly, the temperature  $\tau$  is constant along an integral curve of  $u^a$ , and that  $K^a$  is a Killing vector field.  $\tau$  is the temperature in the rest frame of the fluid.

# Stationary axisymmetric perfect fluid solution

We model a rigidly rotating star surrounded by asymptotically flat vacuum. Metric is of the form

$$g = g_{ab}dx^a dx^b = -Vdt^2 + 2Wdtd\phi + Xd\phi^2 + \bar{g} \quad (1)$$

where  $\bar{g} = e^{2\mu}(d\rho^2 + Zdz^2)$ ,  $\rho = \sqrt{VX + W^2}$ , and  $V, W, X, \mu, Z$  are functions of  $\rho$  and  $z$ .  $X = 0$  on the axis and  $X > 0$  elsewhere.

Angular velocity of “the dragging of inertial frame” is  $\omega = -W/X$ .

Energy-momentum tensor of perfect fluid is

$$T_{ab} = (\epsilon + p)u_a u_b + pg_{ab}, \text{ where fluid's (normalized) 4-velocity is}$$
$$u^a \frac{\partial}{\partial x^a} = \tau K^a \frac{\partial}{\partial x^a}.$$

If we write

$$K^a \frac{\partial}{\partial x^a} = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \quad (2)$$

then rigid rotation corresponds to a constant  $\Omega$  inside the star.





# Tolman's equation

The Killing vector  $K^a$  then extends outside the star to the whole of the stationary axisymmetric vacuum exterior. Inside the star  $\tau^{-1}$  is the redshift factor so that

$$\tau = + \left( V - 2W\Omega - X\Omega^2 \right)^{-1/2} \quad (3)$$



# Tolman's equation

The Killing vector  $K^a$  then extends outside the star to the whole of the stationary axisymmetric vacuum exterior. Inside the star  $\tau^{-1}$  is the redshift factor so that

$$\tau = + \left( V - 2W\Omega - X\Omega^2 \right)^{-1/2} \quad (3)$$

The above equation (modulo a constant fixing the unit of the temperature) is a generalization of Tolman's equation

$\tau \sqrt{-g^{00}} = \text{constant}$  for a fluid at rest.



# Tolman's equation

The Killing vector  $K^a$  then extends outside the star to the whole of the stationary axisymmetric vacuum exterior. Inside the star  $\tau^{-1}$  is the redshift factor so that

$$\tau = + \left( V - 2W\Omega - X\Omega^2 \right)^{-1/2} \quad (3)$$

In the exterior we shall insist on the definition of  $\tau$  using Eq. (3) with the same constant  $\Omega$ . We shall also include an extra factor  $k$  in the RHS.



# Tolman's equation

The Killing vector  $K^a$  then extends outside the star to the whole of the stationary axisymmetric vacuum exterior. Inside the star  $\tau^{-1}$  is the redshift factor so that

$$\tau = + \left( V - 2W\Omega - X\Omega^2 \right)^{-1/2} \quad (3)$$

In the exterior we shall insist on the definition of  $\tau$  using Eq. (3) with the same constant  $\Omega$ . We shall also include an extra factor  $k$  in the RHS.

One has  $K^a K_a = g \left( \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}, \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \right) = -V + 2W\Omega + X\Omega^2$ . In the fluid this becomes  $K^a K_a = -\tau^{-2} \Leftrightarrow u^a u_a = -1$ .



# Tolman's equation

The Killing vector  $K^a$  then extends outside the star to the whole of the stationary axisymmetric vacuum exterior. Inside the star  $\tau^{-1}$  is the redshift factor so that

$$\tau = + \left( V - 2W\Omega - X\Omega^2 \right)^{-1/2} \quad (3)$$

In the exterior we shall insist on the definition of  $\tau$  using Eq. (3) with the same constant  $\Omega$ . We shall also include an extra factor  $k$  in the RHS.

One has  $K^a K_a = g \left( \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}, \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \right) = -V + 2W\Omega + X\Omega^2$ . In the fluid this becomes  $K^a K_a = -\tau^{-2} \Leftrightarrow u^a u_a = -1$ .

If the fluid extends to infinity asymptotic flatness condition that  $V \rightarrow 1$ ,  $W \rightarrow 0$  and  $X = O(\rho^2)$  give  $\Omega = 0$ .



# Tolman's equation

The Killing vector  $K^a$  then extends outside the star to the whole of the stationary axisymmetric vacuum exterior. Inside the star  $\tau^{-1}$  is the redshift factor so that

$$\tau = + \left( V - 2W\Omega - X\Omega^2 \right)^{-1/2} \quad (3)$$

In the exterior we shall insist on the definition of  $\tau$  using Eq. (3) with the same constant  $\Omega$ . We shall also include an extra factor  $k$  in the RHS.

One has  $K^a K_a = g \left( \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}, \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi} \right) = -V + 2W\Omega + X\Omega^2$ . In the fluid this becomes  $K^a K_a = -\tau^{-2} \Leftrightarrow u^a u_a = -1$ .

$\Omega$  is called the angular velocity of the fluid element measured by a stationary observer in the asymptotically flat region in his rest frame.



# g-temperature in the exterior defined

In the exterior we can call  $\tau$  defined by Eq. (3) a temperature or to be careful g-temperature of the vacuum. Here g stands for the gravity. g-temperature becomes imaginary if  $K^a$  becomes spacelike.



# g-temperature in the exterior defined

In the exterior we can call  $\tau$  defined by Eq. (3) a temperature or to be careful g-temperature of the vacuum. Here g stands for the gravity. g-temperature becomes imaginary if  $K^a$  becomes spacelike.

Is g-temperature a mere mathematical construction?



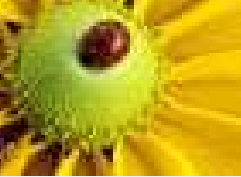


# g-temperature in the exterior defined

In the exterior we can call  $\tau$  defined by Eq. (3) a temperature or to be careful g-temperature of the vacuum. Here g stands for the gravity. g-temperature becomes imaginary if  $K^a$  becomes spacelike.

Is g-temperature a mere mathematical construction?

For any constant  $\beta$ ,  $\frac{\partial}{\partial t} + \beta \frac{\partial}{\partial \phi}$  is a Killing vector field of the spacetime metric of Eq. (1). It is only that in the exterior we choose a  $\beta$  which has a meaning on the surface of the star. Since we are overlooking how the star and the exterior came to be attached, we are overlooking the history of this vector field. But that should not belittle the fact that this vector field is special.



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.

We suppose that we are in the asymptotic tail end far from the singularity in Eq. (3) so that we can consider locally the temperature to be constant.



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.

Let us imagine a test particle in the exterior vacuum at a place with imaginary  $g$ -temperature like a suspended molecule undergoing Brownian motion in a fluid in the limiting theory that the concentration of the suspended particles is small.



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.

Let us imagine a test particle in the exterior vacuum at a place with imaginary g-temperature like a suspended molecule undergoing Brownian motion in a fluid in the limiting theory that the concentration of the suspended particles is small.

Assuming roughly that the g-temperature is constant, and assuming it is measured in kelvin we take Einstein relation in the form  $D = k_B b \tau$  where  $k_B$  is the Boltzmann constant.



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.

Let us imagine a test particle in the exterior vacuum at a place with imaginary g-temperature like a suspended molecule undergoing Brownian motion in a fluid in the limiting theory that the concentration of the suspended particles is small.

Assuming roughly that the g-temperature is constant, and assuming it is measured in kelvin we take Einstein relation in the form  $D = k_B b \tau$  where  $k_B$  is the Boltzmann constant.

Such a simple model however does not give a comparable value of the Planck's constant for the Schrödinger type equation unless we manipulate the factor  $k$  we want to put in the Tolman-Thorne equation Eq. (3).



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.

Let us imagine a test particle in the exterior vacuum at a place with imaginary  $g$ -temperature like a suspended molecule undergoing Brownian motion in a fluid in the limiting theory that the concentration of the suspended particles is small.

Assuming roughly that the  $g$ -temperature is constant, and assuming it is measured in kelvin we take Einstein relation in the form  $D = k_B b \tau$  where  $k_B$  is the Boltzmann constant.

An effect of a variable  $k$  can also be produced by not assuming a linear dependence of the diffusion coefficient on temperature.



# Diffusion coefficient and Einstein relation

Imaginary temperature would create a Schrödinger type equation for a test particle.

Let us imagine a test particle in the exterior vacuum at a place with imaginary g-temperature like a suspended molecule undergoing Brownian motion in a fluid in the limiting theory that the concentration of the suspended particles is small.

Assuming roughly that the g-temperature is constant, and assuming it is measured in kelvin we take Einstein relation in the form  $D = k_B b \tau$  where  $k_B$  is the Boltzmann constant.

We shall take the diffusion coefficient to be  $D = ik_B b |\tau|^{1+\alpha}$ .





# Schrödinger type equation

Assuming  $b$  to be real we write  $D = i\widehat{D}$ . Then in a geodesic normal coordinate the concentration  $C(x, t)$  about a point satisfies the equation

$$\frac{\partial C}{\partial t} = \widehat{D}i\Delta_{\widehat{g}}C - v \cdot \nabla c + \widehat{D}i\Gamma O(|x|^2)C \quad (4)$$

where  $\Gamma$  depends on the curvature components of the Riemannian 3-metric  $\widehat{g}$  at the origin of the coordinate system.



# Schrödinger type equation

Assuming  $b$  to be real we write  $D = i\widehat{D}$ . Then in a geodesic normal coordinate the concentration  $C(x, t)$  about a point satisfies the equation

$$\frac{\partial C}{\partial t} = \widehat{D}i\Delta_{\widehat{g}}C - v \cdot \nabla c + \widehat{D}i\Gamma O(|x|^2)C \quad (4)$$

where  $\Gamma$  depends on the curvature components of the Riemannian 3-metric  $\widehat{g}$  at the origin of the coordinate system.

We assume that the drift  $v = 0$ . Then Eq. (4) gives

$$i\frac{\partial C}{\partial t} = -\widehat{D}\Delta_{\widehat{g}}C - \widehat{D}\Gamma O(|x|^2)C. \quad (5)$$



# Schrödinger type equation

Assuming  $b$  to be real we write  $D = i\widehat{D}$ . Then in a geodesic normal coordinate the concentration  $C(x, t)$  about a point satisfies the equation

$$\frac{\partial C}{\partial t} = \widehat{D}i\Delta_{\widehat{g}}C - v \cdot \nabla C + \widehat{D}i\Gamma O(|x|^2)C \quad (4)$$

where  $\Gamma$  depends on the curvature components of the Riemannian 3-metric  $\widehat{g}$  at the origin of the coordinate system.

We assume that the drift  $v = 0$ . Then Eq. (4) gives

$$i\frac{\partial C}{\partial t} = -\widehat{D}\Delta_{\widehat{g}}C - \widehat{D}\Gamma O(|x|^2)C. \quad (5)$$

This is a Schrödinger type equation for complex  $C$  when  $\widehat{D} > 0$  or its complex conjugate  $C^*$  when  $\widehat{D} < 0$ .



# Schrödinger type equation

Assuming  $b$  to be real we write  $D = i\widehat{D}$ . Then in a geodesic normal coordinate the concentration  $C(x, t)$  about a point satisfies the equation

$$\frac{\partial C}{\partial t} = \widehat{D}i\Delta_{\widehat{g}}C - v \cdot \nabla C + \widehat{D}i\Gamma O(|x|^2)C \quad (4)$$

where  $\Gamma$  depends on the curvature components of the Riemannian 3-metric  $\widehat{g}$  at the origin of the coordinate system.

We assume that the drift  $v = 0$ . Then Eq. (4) gives

$$i\frac{\partial C}{\partial t} = -\widehat{D}\Delta_{\widehat{g}}C - \widehat{D}\Gamma O(|x|^2)C. \quad (5)$$

This is a Schrödinger type equation for complex  $C$  when  $\widehat{D} > 0$  or its complex conjugate  $C^*$  when  $\widehat{D} < 0$ .

Direction of time is given by the sense of rotation.



# Estimating the mobility $b$

We find an estimate for the mobility using Newtonian mechanics. Let the mass of the star be  $M$  and the asymptotically defined angular momentum per unit mass be  $a$ . Since gravitational force is the only force involved assuming Pascal's law we equate the speed  $\omega r$  with  $|b|$  times the gravitational force  $mM/r^2$  and use  $\omega = -W/X \approx 2Ma/r^3$  for large  $r$ .



# Estimating the mobility $b$

We find an estimate for the mobility using Newtonian mechanics. Let the mass of the star be  $M$  and the asymptotically defined angular momentum per unit mass be  $a$ . Since gravitational force is the only force involved assuming Pascal's law we equate the speed  $\omega r$  with  $|b|$  times the gravitational force  $mM/r^2$  and use  $\omega = -W/X \approx 2Ma/r^3$  for large  $r$ .

Heuristically for a test particle of mass  $m$ ,  $|b| = \frac{2|a|}{m}$ . Since for large  $r$ ,  $\tau = -i(|\Omega|r)^{-1} + O(r^{-2})$  we get

$$\widehat{D} \approx -\frac{2ak_{\text{B}}}{m(\Omega r)^{1+\alpha}} \quad (6)$$

We take  $\widehat{D} < 0$ .  $b$  is negative because drag force opposes the velocity.



# Planck's type constant from g-temperature

Equating  $\widehat{D}$  to  $-\frac{\hbar_g}{2m}$  we then get an analogue of the reduced Planck's constant for our imaginary g-temperature:

$$\hbar_g \approx \frac{4ak_B}{(\Omega r)^{1+\alpha}} \quad (7)$$



# Planck's type constant from g-temperature

Equating  $\widehat{D}$  to  $-\frac{\hbar_g}{2m}$  we then get an analogue of the reduced Planck's constant for our imaginary g-temperature:

$$\hbar_g \approx \frac{4ak_B}{(\Omega r)^{1+\alpha}} \quad (7)$$

For numerical estimation we remove  $a$  using the radius of the star:  $a \approx (2/5)R^2\Omega$ . Also use  $\Omega^2 \approx (4\pi/3)\epsilon_{\text{ave}}$  where  $\epsilon_{\text{ave}}$  be the average density. Thus Eq. (7) gives

$$\hbar_g \approx \frac{1.6R^2k_B}{(4\pi\epsilon_{\text{ave}}/3)^{\alpha/2}r^{1+\alpha}} \approx \frac{1.6R^{2+1.5\alpha}k_B}{M^{\alpha/2}r^{1+\alpha}} \quad (8)$$



# Planck's type constant from g-temperature

Equating  $\widehat{D}$  to  $-\frac{\hbar_g}{2m}$  we then get an analogue of the reduced Planck's constant for our imaginary g-temperature:

$$\hbar_g \approx \frac{4ak_B}{(\Omega r)^{1+\alpha}} \quad (7)$$

For numerical estimation we remove  $a$  using the radius of the star:  $a \approx (2/5)R^2\Omega$ . Also use  $\Omega^2 \approx (4\pi/3)\epsilon_{\text{ave}}$  where  $\epsilon_{\text{ave}}$  be the average density. Thus Eq. (7) gives

$$\hbar_g \approx \frac{1.6R^2k_B}{(4\pi\epsilon_{\text{ave}}/3)^{\alpha/2}r^{1+\alpha}} \approx \frac{1.6R^{2+1.5\alpha}k_B}{M^{\alpha/2}r^{1+\alpha}} \quad (8)$$

With  $\alpha = 1$ ,  $R = R_\odot = 6.96 \times 10^{10}$ cm and

$\epsilon_{\text{ave}} = \epsilon_\odot = 1.05 \times 10^{-28}$ cm<sup>-2</sup> we get  $\hbar_g \approx \hbar$  at  $r \approx 10^{18}$ cm.



$r$  distance from the axis of symmetry.

In the equation

$$\hbar_g \approx \frac{1.6R^2 k_B}{(4\pi\epsilon_{ave}/3)^{\alpha/2} r^{1+\alpha}} \approx \frac{1.6R^{2+1.5\alpha} k_B}{M^{\alpha/2} r^{1+\alpha}} \quad (9)$$

$r$  is the distance from the axis of symmetry. On the axis  $g$ -temperature never becomes imaginary because on the axis  $X = 0$ .



## $r$ distance from the axis of symmetry.

In the equation

$$\hbar_g \approx \frac{1.6R^2 k_B}{(4\pi\epsilon_{ave}/3)^{\alpha/2} r^{1+\alpha}} \approx \frac{1.6R^{2+1.5\alpha} k_B}{M^{\alpha/2} r^{1+\alpha}} \quad (9)$$

$r$  is the distance from the axis of symmetry. On the axis  $g$ -temperature never becomes imaginary because on the axis  $X = 0$ .

Crudely speaking imaginary temperature comes out at a distance of  $r_i \approx \Omega^{-1}$ . So the imaginary temperature comes out at a distance of  $r_i \approx \epsilon^{-1/2} \approx 10^{14}$  cm from the axis.



## $r$ distance from the axis of symmetry.

In the equation

$$\hbar_g \approx \frac{1.6R^2 k_B}{(4\pi\epsilon_{\text{ave}}/3)^{\alpha/2} r^{1+\alpha}} \approx \frac{1.6R^{2+1.5\alpha} k_B}{M^{\alpha/2} r^{1+\alpha}} \quad (9)$$

$r$  is the distance from the axis of symmetry. On the axis  $g$ -temperature never becomes imaginary because on the axis  $X = 0$ .

Crudely speaking imaginary temperature comes out at a distance of  $r_i \approx \Omega^{-1}$ . So the imaginary temperature comes out at a distance of  $r_i \approx \epsilon^{-1/2} \approx 10^{14}$  cm from the axis.

For a rotating stationary star the points on a  $t = \text{constant}$  hypersurface where the Killing vector field  $K^a$  becomes null is a surface. Let us denote the minimum  $\rho$ -value on the intersection of this surface with the equatorial plane by  $\rho_i$ . In case this minimum value occurs at a point where the asymptotic coordinate system is a valid approximation we denote the corresponding value by  $r_i$ .



# g-Planck parameter for quantum particles

After finding that  $\hbar_g$  does not match  $\hbar$  in isolated astrophysical objects at meaningful distances we consider modeling elementary particles with gravitation. Here we get a surprise.



# g-Planck parameter for quantum particles

After finding that  $\hbar_g$  does not match  $\hbar$  in isolated astrophysical objects at meaningful distances we consider modeling elementary particles with gravitation. Here we get a surprise.

We assume that the Newtonian approximations crudely apply to a neutron and take  $R = \text{neutron radius} = 1.2 \times 10^{-13} \text{cm}$  and  $M = \text{neutron mass} = 1.25 \times 10^{-52} \text{cm}$ .



# g-Planck parameter for quantum particles

After finding that  $\hbar_g$  does not match  $\hbar$  in isolated astrophysical objects at meaningful distances we consider modeling elementary particles with gravitation. Here we get a surprise.

We assume that the Newtonian approximations crudely apply to a neutron and take  $R = \text{neutron radius} = 1.2 \times 10^{-13} \text{cm}$  and  $M = \text{neutron mass} = 1.25 \times 10^{-52} \text{cm}$ .

First we estimate the distance at which g-temperature becomes imaginary. If we continue to believe that this distance is of the order of  $\Omega^{-1}$  and angular frequency of the neutron is related to its spin  $\frac{1}{2}$  by  $\Omega = 2\hbar M^{-1} R^{-2}$  then this distance is approximately  $0.6 \times 10^{-4}$  Bohr radius.



# g-Planck parameter for quantum particles

After finding that  $\hbar_g$  does not match  $\hbar$  in isolated astrophysical objects at meaningful distances we consider modeling elementary particles with gravitation. Here we get a surprise.

We assume that the Newtonian approximations crudely apply to a neutron and take  $R = \text{neutron radius} = 1.2 \times 10^{-13} \text{cm}$  and  $M = \text{neutron mass} = 1.25 \times 10^{-52} \text{cm}$ .

First we estimate the distance at which g-temperature becomes imaginary. If we continue to believe that this distance is of the order of  $\Omega^{-1}$  and angular frequency of the neutron is related to its spin  $\frac{1}{2}$  by  $\Omega = 2\hbar M^{-1} R^{-2}$  then this distance is approximately  $0.6 \times 10^{-4}$  Bohr radius.

We estimate  $\hbar_g$  for the case  $\alpha = 1$ . Eq. (9) gives  $\hbar_g \approx \hbar$  at  $r \approx 10^{-9} \text{cm} \approx 0.2$  Bohr radius.





# Consequences of variable $h_g$

- If  $h$  occurs in the coupling constant in a theory, then it should be replaced by a variable  $h_g$ .



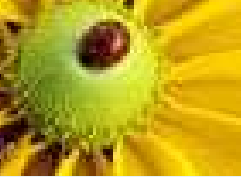
# Consequences of variable $h_g$

- If  $h$  occurs in the coupling constant in a theory, then it should be replaced by a variable  $h_g$ .
- Variable coupling “constant” would give a scalar field with EYM equations.

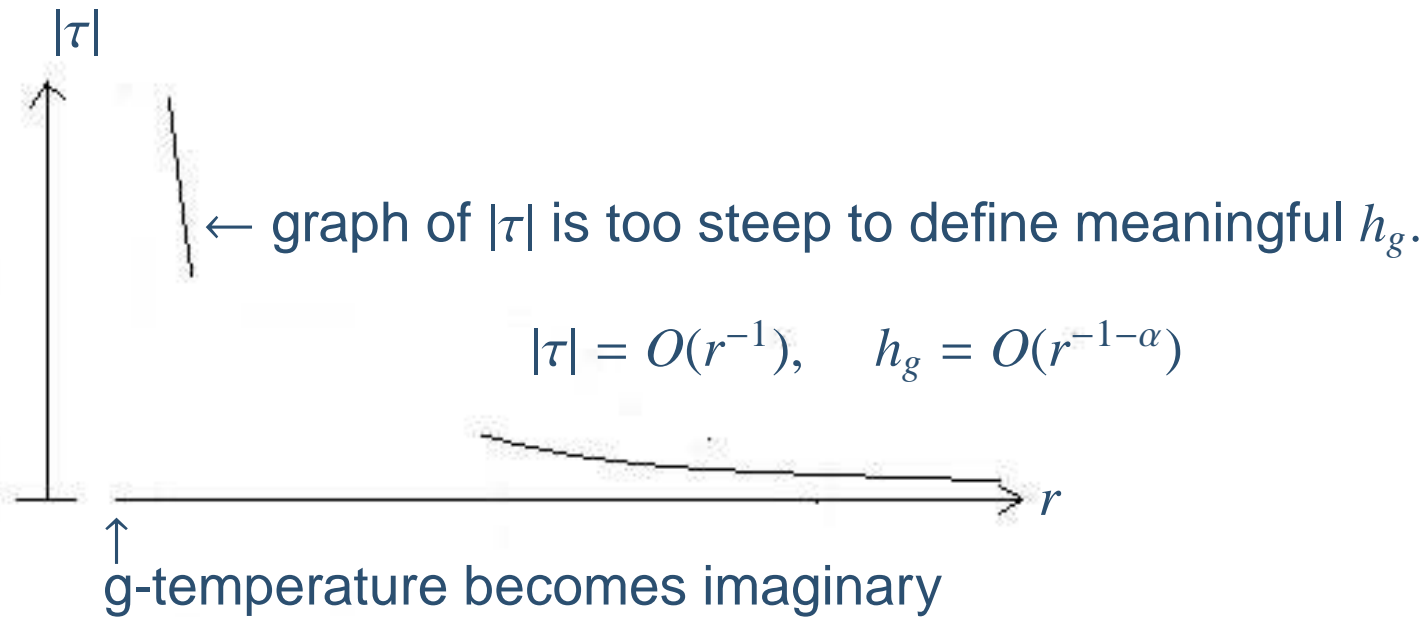


# Consequences of variable $h_g$

- If  $h$  occurs in the coupling constant in a theory, then it should be replaced by a variable  $h_g$ .
- Variable coupling “constant” would give a scalar field with EYM equations.
- Can we get stable particle-like solutions of EYM equations this way?



# A rough graphical representation





# REFERENCES

- Lindblom, L.: "Stationary Stars are Axisymmetric," *The Astrophysical Journal* **208** (1976) 873-880.
- Tolman, R.C.: p313 in *Relativity, Thermodynamics, and Cosmology*, Clarendon Press, Oxford (1934).
- Thorne, K.S.: "The general relativistic theory of stellar structure and dynamics," in C. DeWitt, E. Schatzman and P. Véron ed. *High Energy Astrophysics Vol 3* (1967) 259-441.
- Bekenstein, J.D.: "Black Holes and Entropy," *Phys. Rev. D* **7** (1973) 2333-2346.
- Hawking, S.W.: "Particle Creation by Black Holes," *Commun Math. Phys.* **43** (1975) 199-220.
- Wald, R.M.: "Black hole entropy is the Noether charge," *Phys. Rev. D* **48** (1993) R3427-R3431.
- Jacobson, T.: "Gravitation and vacuum entanglement entropy," arXiv:1204.6349v1 [gr-qc] (2012).



## REFERENCES cont.

- Jacobson, T.: "Thermodynamics of Spacetime: The Einstein Equation of State," *Phys. Rev. Lett.* **75** (1995) 1260-.
- Landau, L.D., Lifshitz, E.M.: *Fluid Mechanics*, English tr. 2nd ed. (1984).
- Bartnik, R., McKinnon, J.: "Particlelike Solutions of the Einstein-Yang-Mills Equations," *Phys. Rev. Lett.* **61** (1988) 141-144.
- Finster, F., Smoller, J., Yau, S.-T.: "The Coupling of Gravity to Spin and Electromagnetism," *Mod. Phys. Lett.* **A14** (1999) 1053-1057.
- van der Bij, J.J., Radu, E.: "On rotating regular nonabelian solutions," *Int. J. Mod. Phys.* **A17** (2002) 1477-1486.
- Yoshida, S., Eriguchi, Y.: "Rotating boson stars in general relativity," *Phys Rev D* **56** (1997) 762-771.
- Heilig, U.: "On the Existence of Rotating Stars in General Relativity," *Commun. Math. Phys.* **166** (1995) 457-493.



## REFERENCES cont.

- MacCallum, M.A.H., Mars, M., Vera, R.: “Stationary axisymmetric exteriors for perturbations of isolated bodies in general relativity, to second order,” *Phys Rev D* **75** (2007) 024017(1-19).
- Pachon, L.A., Rueda, J.A., Sanabria-Gomez, V.J.D.: “Realistic exact solution for the exterior field of a rotating neutron star,” *Phys. Rev. D* **73** (2006) 104038(1-12).
- Pappas, G., Apostolatos, T.A.: “An all-purpose metric for the exterior of any kind of rotating neutron star,” *MNRAS* **429** (2013) 3007-3024.
- Wald, R.: “Black hole in a uniform magnetic field,” *Phys Rev D* **10** (1974) 1680-1685.