Einstein Equations, Imaginary g-Temperature and Quantum Physics

A.K.M. Masood-ul-Alam

We reply some questions raised and some more that could be raised against the method of imaginary g-temperature derived from Einstein equations for uniting general relativity with quantum mechanics.

Introduction

More than a year ago we proposed an imaginary temperature in the outside of a rigidly spinning fluid body in a stationary axisymmetric spacetime [1]. This temperature is formally the same as the temperature inside the body in the sense that it is inversely proportional to the magnitude of a Killing 4-vector which is formally the extension in the exterior vacuum of the same Killing field that produces the temperature inside. The temperature being imaginary it gives a possible route to unify general relativity (GR) and quantum mechanics (QM). Although it is difficult to gauge whether anybody is inspired in anything by our paper through unacknowledged inspiration, we can frankly say that the paper was disregarded by experts. Yet so many different and complicated ideas of physics, as far as we know, rarely came so close in a single thread of thought. The approach does not need any wild speculation of higher dimensional fundamental physics. It continues to distinguish time from the space coordinates. Similarly the approach does not need supersymmetry at a fundamental level although the possibility of an effective theory of parastatistics is there. In addition to clarifying some issues, we demonstrate the richness of this imaginary g-temperature mine and list many solvable open problems. Some of the questions replied have been actually asked. Others I made up myself guessing the trend. Almost all questions replied concern directly material in [1]. Couple of them may concern material in previous replies.

Questions and Replies

Question 1. *"There is no sense in which the vacuum outside the rotating body is co-rotating with the body, nor is there any sense in which it is in equilibrium with* *the body. Hence there is no reason to analytically (sic) extend the temperature in this fashion."*

Reply: The star and the vacuum, that is the whole spacetime, is in stationary equilibrium and that is the only complete sense of equilibrium. The sense of "thermal equilibrium" can be given only after defining the pure gravitational/spacetime entropy density. Although I have not yet defined it, I defined a pointwise temperature. Some reasons for not defining the pure gravitational entropy at this stage is given below. But these concerns are immaterial for this question. The main point is that the Killing vector field exists outside the star and that is what we are using to define g-temperature. Readers of [1] were warned (top line on page 3) that physically the temperature is not defined outside the star. As in the Kerr solution metric part $Wdtd\phi$ is not zero in the vacuum. Thus the vacuum is rotating.

Question 2. *According to your definition Minkowski spacetime in the standard coordinates has a nonzero g-temperature. This is wrong.*

Reply: It is not wrong. This temperature is the perfect number 1 except that in [1] we also kept the possibility of a parameter k in front of the inverse redshift factor. In fact that Minkowski spacetime in standard coordinates has constant temperature 1 can be considered a plus point of g-temperature. Why should we expect it to be zero when Minkowski spacetime has time and finite maximum speed? If imaginary time corresponds to real temperature, a possibility is there that real time has something to do with complex temperature. Besides ordinary temperature is due to motion and time is a parameter of motion. Thus it is not strange that adding time to three space would introduce temperature. The formula for the Unruh temperature of the Minkowski spacetime in Rindler coordinates gives zero temperature in standard coordinates. g-temperature is not Unruh temperature. We are not considering Minkowski spacetime with photons in it.

Question 3. *Before becoming imaginary your temperature is becoming infinite. Do we see infinity in nature?*

Reply: This is connected to the Killing vector field becoming null. First, such a situation is well-known in the detector-quantum field interaction (Deser and Levin [2]) and is usually related to the onset of the quantum effect which we are suspecting to be of gravitational origin. Second, we also encounter infinite temperature in the study of negative temperature in solid state physics where negative temperature is higher than infinite temperature. Thus this question is not a valid objection to the proposed g-temperature. It disappears when temperature is explained from the functional relation of entropy density with the energy density.

Question 4. *While calculating the mobility you equated centripetal force with the tangential velocity which are perpendicular not antiparallel.*

Reply: When considering pressure in the host fluid in which the Brownian motion is taking place what is wrong in assuming Pascal's law as a first approximation or first trial? The penalty for this heuristic explanation is possibly the reason we needed to use the specific heat from after the hump of the Schottky anomaly for a better match of g-Planck's constant with the actual Planck's constant. Again I think for a more rigorous treatment we have to wait for the pure gravitational/spacetime entropy to be defined.

Question 5. *In the vacuum the pressure and mass-energy density is zero. Why there would be pressure in the host fluid?*

Reply: This is a sort of "osmotic" pressure. I am not sure but it can possibly be considered as an additive part of this zero pressure. The equation for hydrostatic equilibrium (contracted Bianchi identity) is trivially satisfied in the vacuum since mass-energy density ρ and pressure p vanish. Since this equation is linear in ρ and p, the gradient of this "osmotic" pressure is then proportional to the gradient of the "gravitational potential." One question is whether the Schrödinger-type equation and the mobility can be derived from the pure gravitational/spacetime entropic pressure and density via the Bianchi identity.

Question 6. *Nelson* [3] *used Brownian motion approach of explaining QM from classical mechanics and that now developed into Fényes-Nelson stochastic formulation of QM. Can you compare your idea.*

Reply: Four important differences are as follows. First, we are trying to derive QM rather than using stochastic formulation of Newtoninan mechanics to mimic QM. Thus our formulation, though still incomplete presently, will not contradict QM in the domain where it is applicable. In this respect we are nearer to the approach of Comisar [4] which approach involves imaginary diffusion coefficient (first done by Fürth [5]). Second, instead of simply hypothesizing a new theory we derived the imaginary nature of the diffusion constant from Einstein equations. Third, our method has potential for tackling other quantum concepts such as the so-called spatial quantization, antiferromagnetism and so forth. Fourth, since our starting point is Einstein equations, slight causality violation is not a contradiction. It will take sometime before we can present our ideas as clear as in Nelson [3]. Our scope is difficult and narrow as we are interested in unification.

Question 7. *How does your approach differ from that of Jacobson et. al. and others following them?*

Reply: Essentially there are two opposite implications working.

One implication involves entropic gravitation which explains the origin of gravity from quantum physics by way of entropy. This method originated from Sakharov's metric elasticity method [6] to derive classical GR from quantum gravity. Sakharov was inspired by the work of Zel'dovich on the effect of vacuum quantum flactuations on the cosmological constant. The method has been furthur explored by Adler [7] and Hu [8]. Finally Jacobson [9],[10] made a significant breakthrough and was followed by Frampton and Karl [11] to name few authors. This method is persistent in giving importance to particle physics (and/or black holes) and quantum physics as the source of gravitation. Roughly speaking we can denote this implication QM $\Rightarrow$ GR.

The other implication involves pure gravitational/spacetime entropy and claims usual quantum effects of physics to be of gravitational origin. We have to add the word pure to stress that in classical physics entropy due to gravitation is considered mechanical. An example of pure gravitation/spacetime entropy, more general than black hole entropy, has been considered, for example, in Gibbons [12]. In the second implication spacetime with the metric is everything; the energy-momentum tensor is determined by the metric via Einstein equations. The so-called matter fields if needed should be extracted from the energy-momentum tensor and (if necessary) from the differential Bianchi identity. These matter fields are patterns in the field of the energy-momentum tensor. At present they are put in the Lagrangian guessed from experiments. From the Lagrangian one gets the field equations for the matter coupled with Einstein equations. A by-product of this method is that matter fields are given, at least, equal footing as gravity and are often considered sources of the gravity. We read results of experiments by the transfer of energy and momentum and study wave functions by diffraction patterns. Thus one suspects that matter fields, as far as they are physically relevant, can possibly be extracted by noting the local thermodynamic and hydrodynamic quantities derived from the energy-momentum tensor and asymptotic constants obtained by integrating by parts certain divergence forms involving these quantities. Some asymptotic constants may appear as decay constants in the metric. Although the idea that matter is "bent space" is old, the effort to derive the matter fields from the energy-momentum tensor has not been taken seriously until recently. Besides the old pursuit was philosophical and was burdened with the vague concepts of inertia and Mach's principle. An interesting discussion on the old pursuit can be found in Lynden-Bell [13] and some references therein.

In short, we see two opposite implications in the two approaches. Of course no pursuit is totally futile. As we are approaching unification, ideas seem to converge. Some open problems are to re-interpret the correct results of the first implication by the second implication. These two implications are not naturally symmetric. For example, there are many nontrivial empty asymptotically simple spacetimes. These are diffeomorphic to $\mathbb{R} \times \mathbb{R}^3$ and globally regular. See Corvino [15]. Existence of many such spacetimes was conjectured earlier by Penrose [16]. But there is no particle without gravitation. Also since the energy-momentum tensor cannot be separated as part due to matter fields and part due to gravity and as it may contain the metric, it does little sense to consider it as the source of gravity.

Question 8. *"...if anything were co-rotating in the space-like region, it would be moving faster than light, and there is probably no sensible way to place a tachyonic system in equilibrium with a non-tachyonic one."*

Reply: The spacetime is the star surrounded by vacuum. There is no "anything" in the vacuum. The temperature is imaginary. The Killing vector field is defined in the vacuum. Tachyonic system is totally uncalled for here.

Question 9. *Why do you say the reference to tachyonic system is totally uncalled for?*

Reply: There could be some causality violation because we have essentially a wave function satisfying a Schrödinger-type equation. But our basis is Einstein equations and we do not have tachyon in the theory. The reference to tachyonic system is against the spirit of my paper where I am trying to explain origin of physical quantum phenomena with Einstein's theory of gravitation. I also see the extrapolation of the particle concept in the idea of tachyon. Theory of Einstein equations is an exact theory whereas particle concept is a vague concept originating from the symmetry of the Minkowsky spacetime.

We think our lack of progress in unification is partly due to our preoccupation with matter fields and the particle concept. The name of the subject might have changed to high energy physics but the preoccupation persists. I am not saying this because of funding allocation, job search issues or good students going to only one field (although livelihood and opportunities for all research fields should also be a valid concern). I shall take this opportunity to comment on the wild speculation mentioned in the introduction. If one considers a physical field to correspond to an extra dimension for mathematical convenience that is another thing. We believe in Einstein-matter-fields coupled equations. But theories trying to explain the symmetries of non-gravitational interactions from higher dimensional spacetime geometry which started from about 1960 has no experimental basis. It is driven by a belief of aesthetic nature. Mathematics is certainly richer in higher dimensions. But then grounded on Bochner's myth, limited by Gödel's sky and in the middle with the trunk of von Neumann's elephant, what one cannot do in 10-dimension?

Question 10. *You correctly point out "a large number of difficulties with the proposal of identifying this imaginary temperature somehow with Planck's constant (sic!), yet" provide "no viable starting point for how to circumvent those problems.*

Reply: Viable starting point was the main point of the paper: introduction of the imaginary g-temperature. Wide circulation of this idea was necessary so that many minds could have reflected on the issue. My paper claimed that the Planck's constant of the QM in the Minkowski spacetime is actually of general relativistic origin and the actual entity behind it is not a constant at all in a general spacetime. This actual entity behaves like a constant in the presence of and near a "quantum particle" or more precisely, in a gravitational field of appropriate mass-energy density and "angular momentum density."

Question 11. *"I think the author is mistaken in saying that the temperature he is discussing is not the Bekenstein temperature in the case of a black hole. He correctly points out that the temperature would be infinite at the horizon. This refers to the co-moving temperature, i.e. the temperature in a static frame at the horizon, which is indeed divergent. The finite Bekenstein (Hawking) temperature refers instead to the temperature in the static frame at infinity."*

Reply: We see in this criticism an indirect admission that the g-temperature proposed is more fundamental than that of "Bekenstein (Hawking)" temperature. If the temperature that needs Hartle-Hawking states and quantum theory to be justified, can be inferred from classical GR without quantum theory then does not that suggest our proposal to be of deeper importance? Bekenstein temperature was defined only on the horizon and g-temperature is undefined there. We do not equate two infinities particularly since we kept the possibility of a parameter.

Question 12. *Explain further on your statement in* [1] *namely "except for Plancks constant, quantum mechanics is of mathematical origin...."*

Reply: Basic QM is Fourier analysis. Its touch with physics lies in the fact that the physical quantity called linear momentum is proportional to the Fourier conjugate variable and the proportionality constant is the reduced Plancks constant. This may have some connection with the theory of random flights. One recalls a saying of Professor Chandrasekhar (p9, [17]): "And Markoff's procedure illustrates a very general principle that it is the Fourier transform of the probability function, ..., that has a more direct relation to the physical situations." It is only recently some researchers, though still very few, have challenged the traditional meaning of time. One can suspect the statistical connection of time. Once we have the correct form of the pure gravitational/spacetime entropy, we may be able to understand the basis of this proportionality better via the correct form of the diffusion equation. While these are future issues, one thing is clear from the method of quantization applied to finance and logic that QM is a general mathematical method. Physicists discovered it first.

Question 13. *Can you elaborate on the issues of the pure gravitational/spacetime entropy hinted in the reply of the first question? Why are you not defining it?*

Reply: There are several candidates for the entropy of a spacetime as a manifold and having metrics with certain degree of differentiability. For example one has the fundamental solution of the heat equation and its role in the construction of Hartle-Hawking states. Breaking two spacetimes (star and vacuum) and patching them has some similarity in breaking and fusing ceramics and solids. Before we choose the appropriate entropy density that can capture all the issues, I want to understand other instances of complex temperature and complex entropy in solid states in particular in antiferromagnetism and spin glass models. I am not in a hurry. I already showed in [1] how Schrödinger-type equations and a Planck-type "constant" may come out from Einstein equations with a remarkable coincidence that distinguishes a classical star from a quantum particle.

Question 14. *Your Schrödinger-type equation is potentially nonlinear. Will that nonlinearity pass objections to Bohm's short-range modifications?*

Reply: At present I shall not be able to quantitatively answer the question. There is nonlinear debris in the present equation derived heuristically and most likely there will be nonlinearity in the more accurate one. However this nonlinearity is like that of Einstein equations in that there is always a normal coordinate neighborhood where it is negligible.

Question 15. *What can you do with imaginary g-temperature?*

Reply: With the Planck-type "constant" of the imaginary g-temperature the vacuum energy estimates can be drastically reduced ([14]). Possible non-uniformity in vacuum energy density could possibly be explained by it. Imaginary g-temperature provide a way of averaging effects of strong gravity. Why certain theory (Euclidean gravity, spacetime bubble and so forth) works in some situations for this purpose can possibly be explained by imaginary g-temperature. In Feynman diagrams the pointwise conservation of the energy-momentum tensor of the Einstein theory is totally sacrificed in favor of particle-like momentum conservation. If we stick to this procedure we have no way to know what goes between the localized packet of energy-momentum tensor and the new born entity that can be called particle. Considering these rules as an averaging procedure that works in certain regime one may try to explain these rules using Einstein equations and the imaginary g-temperature. These rules referred to above are those of Minkowski spacetime but now we have to add gravitation. The new Planck-type "constant" may allow us to do it without the divergences in the Feynman diagrams of the quantum theory of gravitation.

Question 16. *You have not included the time variable yet.*

Reply: Although the derivation of the Schrödinger-type equation is done only for a stationary spacetime, there will be analogous things in a general spacetime. Apart from a general definition of the pure gravitational/spacetime entropy, I cannot see that there would be any exact generalization in closed form that will cover all situations. The closed form or nice fundamental objects are the Einstein equations. Rest are averaging and approximation. Our assumption of stationary spacetime with a global $t = const$ hypersurface is not the same problem as that involved in the definition of vacuum state in curved spacetime QFT. We seek pointwise defined fields not global objects defined on a space of global objects. Next I want to consider a differentially rotating single star (particle) with dissipation. Then I want to consider a multi-particle system.

Question 17. *In what other situations gravitation and quantum effects appear to be related.*

Reply: I list four types of situations where gravitation touched QM with possible implication that GR⇒QM. There could be many more that I did not notice. I do not include the correspondences which are popping up in various type of field theories because at present they do not appear to have no more significance than that one begins with similar Lagrangians or equations and hence gets similar effects. I do not include anything involving gravitons since these are explained using quantum physics from the start and also using the linearized theory of gravity.

Type I. Particle like solutions of EYM or EYMD equations (Bartnik and McKinnon [18] and Finster, Smoller and Yau [19]).

Type II. Effort by Barut, Cruz and Sobouti [20] to construct massive quantum particle as localized packets of solutions of linearized gravitational field equations.

Type III. In a static stellar model the dropping of the adiabatic index below 1.2 corresponds to the neutron drip. This dropping follows from a differential inequality relating the pressure and mass-energy density:

$$5\rho^2 \geq 6p(p+\rho)d\rho/dp$$

[21], Beig and Simon [22] and finally Lindblom and Masood-ul-Alam [23] showed that this inequality has a strong connection with the positive mass theorem of Schoen and Yau [24]. Ever since the coincidence is noted in the conclusion of [21], I am trying to relate the neutron drip with the crack in the graph of the equation of state. An attempt to generalize this inequality for a rotating stellar model led to the discovery of the concept of the imaginary g-temperature.

Type IV. In a static stellar model the conformal factor ψ, that makes the induced metric g on the t=constant surface conformally Euclidean $\psi^4 g$, is the square root of the norm of the spinor satisfying the Dirac-Weyl neutrino equation. As if the probability function of a neutrino is compensating the curvature of the space.

Conclusion

We have shown the importance of imaginary g-temperature. Our arguments strongly suggest a new method of averaging strong gravitational effects using Einstein equations. This method naturally involves a varying Planck-type "constant" that shows its importance in the usual quantum regime. We point to a rich mine of ideas important for unification, mine that is hitherto unexplored. We suggested many doable open problems. Before declaring GR incomplete or daring to replace it we can do lots of things in the framework of GR.

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Mathematical Sciences Center and Department of Mathematical Sciences,
Tsinghua University, Haidian District, Beijing 100084, PRC.
E-mail address: abulm@math.tsinghua.edu.cn