

# On a non-semisimple Lie algebra module and algebraic geometry

An Huang

February 13, 2012

My presentation was titled "On a non-semisimple Lie algebra module and algebraic geometry". This is joint work in progress with Bong Lian and Shing-Tung Yau. First I should apologize that, it was completely due to my fault, that the title was not so clear: I really wanted to mean a non-semisimple module of a reductive Lie algebra.

I discussed very briefly about our program of building the B-model for Calabi-Yau complete intersections in homogeneous space. The so-called tautological system developed by Bong Lian, Ruifang Song and Shing-Tung Yau is expected to play a fundamental role in this program: it is a holonomic D-module which governs the period integrals. The main point of this presentation was about recent progress made on calculating the holonomic rank of this D-module. This problem is practically important in at least two aspects:

1. To answer the question of whether all holomorphic solutions are actually periods.
2. To locate the large complex structure limit points in the complex structure moduli space.

Partially inspired by a paper of Adolphson, we found that a certain non-semisimple module of a reductive Lie algebra plays a very important role in this holonomic rank problem. This non-semisimple module is manageable under certain circumstances, which enables us to prove theorems and make conjectures on the holonomic rank problem.

I reported two theorems.

The first theorem is an explicit result on the holonomic rank in a very special case, which can serve as an evidence that all solutions of the tautological system are periods. A few days after, Spencer Bloch proved an

impressive generalization of this result using a completely different method, which is very encouraging.

The second theorem is again on the holonomic rank in another very special case, which is of interest especially as it tells us a large complex structure limit point for CY hypersurface family in the grassmannian  $G(2, n)$ .

Next I stated a conjecture and one of its possible extensions inspired by these results, which relates Lie algebra homology of this module with singular cohomology of certain affine variety, possibly on the level of Hodge structure. In particular, this conjecture directly implies further results on the large complex structure problem.

Aside from our program on the B-model, there is collaborated research going on centered around this conjecture with Spencer Bloch and Marcel Bokstedt.

During and after my presentation, Lizhen Ji, Eduard Looijenga, Jian Zhou and a few other guys asked me a few questions and corrected some of my words. This was very helpful to me. After the presentation, I further discussed the material with Kefeng Liu.

It is my great pleasure to thank Spencer Bloch, Marcel Bokstedt, Shenghao Sun, Jie Zhou, and Minxian Zhu for very useful discussions. The main part of the work reported was done during my visit at the Tsinghua Mathematical Sciences Center, which provides a great environment.