

On the Compressible Euler Equations coupled with gravitational or electric fields

Tao Luo

1. DYNAMICAL STABILITY OF WHITE DWARFS

A white dwarf is a small star composed mostly of *electron-degenerate* matter which is supported by the degenerate pressure of electrons obeying the following asymptotics:

$$(1.1) \quad \begin{aligned} p(\rho) &= c_1\rho^{4/3} - c_2\rho^{2/3} + \dots, & \rho \rightarrow \infty, \\ p(\rho) &= d_1\rho^{5/3} - d_2\rho^{7/3} + O(\rho^3), & \rho \rightarrow 0, \end{aligned}$$

where c_1, c_2, d_1 and d_2 are positive constants, where ρ is the density. There is an upper limit to the mass of an electron-degenerate object, *the Chandrasekhar limit*, beyond which electron degeneracy pressure in the star's core is insufficient to balance the star's own gravitational self-attraction. This raises a natural question: If the total mass of a white dwarf is beneath the Chandrasekhar limit, is it dynamically stable? In the framework of newtonian mechanics, the evolution of a stars of compressible fluids is modeled by the following Euler-Poisson equations in three spatial dimensions:

$$(1.2) \quad \begin{cases} \rho_t + \nabla \cdot (\rho \mathbf{v}) = 0, \\ (\rho \mathbf{v})_t + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla p(\rho) = -\rho \nabla \Phi, \\ \Delta \Phi = 4\pi G \rho. \end{cases}$$

where ρ is the density, $\mathbf{v} = (v_1, v_2, v_3)$ is the velocity field, p is the pressure, Φ is the gravitational potential given by

$$(1.3) \quad \Phi(x) = -G \int_{\mathbb{R}^3} \frac{\rho(y)}{|x-y|} dy.$$

There are two types of Equilibrium Configurations, rotating and non-rotating star solutions, a non-rotating star solution is a time-independent solution to (1.2) of the form: $(\rho, 0, \Phi)(x)$. A rotating star solution is a time-independent solution with the prescribed angular momentum or angular velocity. We will focus on the non-rotating star solutions here, which satisfy the following system

Variation Formulation: Let $\int =: \int_{\mathbb{R}^3}$. For $f \in L^1(\mathbb{R}^3)$, set $Bf(x) = \int \frac{f(y)}{|x-y|} dy = f * \frac{1}{|x|}$. For $0 < M < +\infty$, let

$$(1.4) \quad \begin{aligned} X_M &= \{ \rho : \mathbb{R}^3 \rightarrow \mathbb{R}, \rho \geq 0, a.e., \int \rho(x) dx = M, \\ &\int [A(\rho(x)) + \frac{1}{2} \rho(x) B\rho(x)] dx < +\infty \}, \end{aligned}$$

For $\rho \in X_M$, define the energy functional H for non-rotating stars by

$$(1.5) \quad H(\rho) = \int [A(\rho(x)) - \frac{1}{2} \rho(x) B\rho(x)] dx.$$

In the following, we concentrate on the study of white dwarfs, i. e.

$$(1.6) \quad \begin{cases} \lim_{\rho \rightarrow 0} p(\rho)/\rho^{4/3} = 0, \lim_{\rho \rightarrow +\infty} p(\rho)/\rho^{4/3} = K < \infty, \\ p'(\rho) > 0, \text{ for } 0 < \rho < +\infty. \end{cases}$$

In [1], a new compactness result was established for the minimizing sequence for the energy functional H when the total mass is beneath a critical mass, by using which, the dynamical stability of non-rotating white dwarf with the total mass beneath the chandrasekhar Limit was established in [1].

Those results were extended to the *rotating* white dwarfs by Smoller and Luo ([1]), for which the analysis is much more complicated, even in the formulation of the problem.

2. EULER EQUATIONS COUPLED WITH ELECTRIC FIELDS

In the hydrodynamical model of semiconductor devices or plasma, the following system of Euler-Poisson equations are used:

$$(2.1) \quad \begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (p(\rho) + \rho u^2)_x = \rho E, \\ E_x = \rho - b(x). \end{cases}$$

where E : the electric field. $b(x) > 0$: the density of fixed, positively charged background ions (doping profile).

We are interested in the existence and dynamical stability of transonic shock solutions for the above system. The steady transonic shock solutions with the boundary conditions :

$$(2.2) \quad (\rho, u, E)(0) = (\rho_l, u_l E_l), \quad (\rho, u_r)(L) = (\rho_r, u_r)$$

satisfying $\rho_l u_l = \rho_r u_r$ are constructed in [3] when $b > 0$ is constant by using the shock matching method. Uniqueness and multiplicity of transonic shock solutions are also discussed in [3] depending on the boundary data and interval length L . The structural stability of the transonic solutions obtained mentioned above was proved in [4] when b is a small perturbation of a constant. Moreover, the dynamical stability conditions of the steady transonic solutions are identified in [4]. A free boundary problem is considered in [4] with the dynamical shock front as the free boundary. The proof in solving this free boundary problem globally in time involves the following ideas:

- 1) Transform the free boundary problem to fixed initial boundary value problem and use the entropy condition on the shock to obtain the boundary dissipation estimates.
- 2) Spectral analysis of the linearized operator to get decay estimates of the linearized problem;
- 3) Nonlinear iteration.

REFERENCES

- [1] T. Luo & J. Smoller, Nonlinear Dynamical Stability of Newtonian Rotating and Non-rotating White Dwarfs and Rotating Supermassive Stars, *Comm. Math. Phys.* 284, 425-457 (2008).
- [2] T. Luo & J. Smoller, Existence and Nonlinear Stability of Rotating Star Solutions of the Compressible Euler-Poisson Equations (with J. Smoller), *Arch. Rational Mech. & Anal.* 191, No.3, 447-496 (2009).
- [3] T. Luo & Z. Xin, Transonic shock solutions for a system of Euler-Poisson Equations, to appear in *Comm. Math. Sci.*
- [4] T. Luo, J. Rauch, C. Xie & Z. Xin, Stability of Transonic Shock Solutions for One-Dimensional Euler-Poisson Equations, *Arch. Rational Mech. & Anal.* Vol. 202, No. 3, P. 787-827 (2011).

CENTER AND DEPARTMENT OF MATHEMATICAL SCIENCES, TSINGHUA UNIVERSITY
E-mail address: tluo@math.tsinghua.edu