Framelet-Based Algorithm for Segmentation of Tubular Structures

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Tight-frame, a generalization of orthogonal wavelets, has been used successfully in various problems in image processing, including inpainting, impulse noise removal, super-resolution image restoration, etc. The talk is on using tight-frame for segmenting tubular structures.

The first part of the talk is about tight-frames. The construction of compactly supported (bi-)orthonormal wavelet bases of arbitrarily high smoothness has been widely studied since Ingrid Daubechies's celebrated works back in late 1980's and early 1990's. Tight frames generalize orthonormal systems and give more flexibility in filter designs. A system $X \subset \mathcal{L}^2(\mathbb{R})$ is called a tight frame of $\mathcal{L}^2(\mathbb{R})$ if

$$\sum_{h\in X} |\langle f, h\rangle|^2 = \|f\|^2,$$

holds for all $f \in \mathcal{L}^2(\mathbb{R})$, where $\langle\cdot,\cdot\rangle$ and $\|\cdot\| = \langle\cdot,\cdot\rangle^{1/2}$ are the inner product and norm of $\mathcal{L}^2(\mathbb{R})$. This is equivalent to

$$\sum_{h\in X} \langle f, h\rangle h = f, \quad f \in \mathcal{L}^2(\mathbb{R}).$$

Hence, like an orthonormal system, one can use the same system X for both the decomposition and reconstruction processes. They preserve the unitary property of the relevant analysis and synthesis operators, while sacrificing the orthonormality and the linear independence of the system in order to get more flexibility.

If X is the collection of dilations of L^j, $j \in \mathbb{Z}$, and shifts of a finite set $\Psi \subset \mathcal{L}^2(\mathbb{R})$, i.e.,

$$X(\Psi) = \{\psi^\ell_{j,k} : \psi \in \Psi, 1 \leq \ell \leq r; j,k \in \mathbb{Z}\}, \tag{1}$$

where $\psi^\ell_{j,k}(t) = L^{j/2}\psi^\ell(L^j \cdot -k)$, then $X(\Psi)$ is called, in general, a wavelet system. When $X(\Psi)$ forms an orthonormal basis of $\mathcal{L}^2(\mathbb{R})$, it is called an orthonormal wavelet system. In this case the elements in Ψ are called the orthonormal wavelets. When $X(\Psi)$ is a tight frame for $\mathcal{L}^2(\mathbb{R})$ and Ψ is generated via a multiresoultaion analysis, then each element of Ψ is called a tight framelet, and $X(\Psi)$ is called a tight framelet system. Tight framelet systems generalize orthonormal wavelet systems.

In the second part of the talk, we illustrate the tight-frame algorithm in one application in image processing– high-resolution image reconstruction (HRIR). HRIR arises in many applications, such as remote sensing, surveillance, and medical imaging. One HRIR model views the passage of the high-resolution image through a blurring kernel built from the tensor product of a univariate low-pass filter of the form $\left[\frac{1}{2}+\epsilon, 1, \cdots, 1, \frac{1}{2}-\epsilon\right]$, where ϵ is the displacement error. When the number L of low-resolution sensors is even, tight frame symmetric framelet filters were constructed from this low-pass filter using the unitary extension principle. The framelet filters do not depend on ϵ, and hence the resulting algorithm reduces to that of the case where $\epsilon = 0$. Furthermore, the framelet method works for symmetric boundary conditions. This greatly simplifies the algorithm. In this talk, we introduced piece-wise linear tight framelets and derive a tight framelet algorithm with symmetric boundary conditions that work for both odd and even L. An analysis of the convergence of the algorithms is also given. The details of the implementations of the algorithm are also given.

The last part of the talk is about segmentation of tubular structure. Segmentation is the process of identifying object outlines within images. There are quite a few efficient algorithms for segmentation that depend on the variational approach and the partial differential equation (PDE) modeling. In this talk, we presented the tight-frame approach to automatically identify tube-like structures such as blood vessels in Magnetic Resonance Angiography (MRA) images. Our method iteratively refines a region that encloses the possible boundary or surface of the vessels. In each iteration, we apply the tight-frame algorithm to denoise and smooth the possible boundary and sharpen the region. We prove the convergence of our algorithm. Numerical experiments on real 2D/3D MRA images demonstrate that our method is very efficient with convergence usually within a few iterations, and it outperforms existing PDE and variational methods as it can extract more tubular objects and fine details in the images.