High Spin Topologically Massive Gravity

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Abstract

We study the high spin fields coupled to topologically massive gravity in AdS_3, paying special attention to the nature of the theory at the critical point. We propose an action incorporating the high spin AdS_3 gravity and the topological Chern-Simons term for high spin fields. We discuss the fluctuation spectrum around the AdS_3 vacuum and find that besides the usual massless modes there are local massive modes.

1 Introduction

There has been a long history on high spin(HS) field theory, since Fierz-Pauli. The free HS theory is well-defined in both flat and curved spacetime but the interacting HS field theory is only well-defined in a spacetime with cosmological constant, positive or negative. It is remarkable that in $D \geq 4$, once we include one massless field with spin higher than two into the interaction, we must include an infinite tower of massless fields with various higher spins and also other compensator fields. This makes the interactive HS field theory very complicated. Nevertheless, the study on HS field theory has drawn much attention in the past decade, for its close relation with string theory and AdS/CFT correspondence.

It has been known for a long time string theory has a rich symmetry. The massless HS fields appear as the excitations of the string in the tensionless limit, i.e. $m^2 \propto \frac{1}{\alpha'} \to 0$. On the other hand, the HS field theory plays an important role in the AdS/CFT correspondence. It was conjectured by Polyakov and Klebanov in 2002 that the singlet sector of three dim. O(N) vector model in the large N limit is dual to bosonic minimal Vasiliev HS theory in AdS_4. This conjecture is very interesting, but will not be the topic I am going to talk about.

In the past one year, the HS field theory in AdS_3 has gained much attention. Strictly speaking, the concept of spin in 3D is ill-defined, as the Poincare group does not allow massless representation of arbitrary spin. Nevertheless, we can still consider symmetric tensor of rank s as "spin"-s field, as widely used in the community. It turns out that the HS theory in AdS_3 is simpler. Another remarkable feature is that there is no local physical d.o.f. for gravity and higher spin fluctuations in AdS_3. However, there could be boundary degrees of freedom.

For pure AdS_3 gravity, it could be written as a Chern-Simons theory. Combining the dreibein and the spin connection into two SL(2,R) gauge potentials:

$$A = (\omega^a_\mu + \frac{1}{l} e^a_\mu) J_a dx^\mu, \quad \tilde{A} = (\omega^a_\mu - \frac{1}{l} e^a_\mu) J_a dx^\mu.$$

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where $\omega^a = \epsilon^a{}_{bc}\omega^{bc}$, the Einstein-Hilbert action with a negative cosmological constant term could be written in terms of Chern-Simons action

$$S_{EH} + S_\Lambda = S_{CS}[A] - S_{CS}[\tilde{A}] \tag{1.1}$$

where with $k = \frac{l}{4G}$

$$S_{CS}[A] = \frac{k}{4\pi}\int \mathrm{Tr}(A \wedge dA + \frac{2}{3}A \wedge A \wedge A). \tag{1.2}$$

To account for spin-3 field, we can generalize the gauge group from SL(2,R) to SL(3,R). The $SL(3,R)$ group has the generators $J_a, T_{ab}(a, b = 1, 2, 3)$ with T_{ab} being symmetric and traceless, satisfying the following commutation relations:

$$[J_a, J_b] = \epsilon_{abc}J^c, \qquad [J_a, T_{bc}] = \epsilon^d{}_{a(b}T_{c)d},$$

$$[T_{ab}, T_{cd}] = \sigma(\eta_{a(c}\epsilon_{d)be} + \eta_{b(c}\epsilon_{d)ae})J^e.$$

We combine the frame-like fields and corresponding connections of spin-2 and spin-3 into two gauge potentials $A, \tilde{A}$

$$\begin{aligned} A &= ((\omega^a_\mu + \frac{1}{l}e^a_\mu)J_a + (\omega^{ab}_\mu + \frac{1}{l}e^{ab}_\mu)T_{ab})dx^\mu, \\ \tilde{A} &= ((\omega^a_\mu - \frac{1}{l}e^a_\mu)J_a + (\omega^{ab}_\mu - \frac{1}{l}e^{ab}_\mu)T_{ab})dx^\mu; \end{aligned}$$

Here e^{ab}_μ is the frame-like field for spin-3 field, and ω^{ab}_μ is corresponding spin-connection. Then the CS action gives a theory for spin-3 field coupled to gravity with a negative cosmological constant. It was found that with generalized Brown-Henneaux b.c., spin-3 gravity in AdS_3 has W_3 asymptotic symmetry algebra, with the same central charge $c_L = c_R = 3l/2G$ as pure gravity. It has been conjectured that for spin-n HS gravity in AdS_3, its asymptotic symmetry algebra is W_n algebra with the same central charges.

To have local gravitational degree of freedom, one may add higher-derivative terms in the action. However, usually it induces ghost and instability. In three dimensions, one simple choice is to add a gravitational Chern-Simons term, which is parity breaking and topological:

$$I_{CS} = \frac{1}{2\mu}\int d^3x\sqrt{-g}\varepsilon^{\lambda\mu\nu}\Gamma^\rho_{\lambda\sigma}\left(\partial_\mu\Gamma^\sigma_{\rho\nu} + \frac{2}{3}\Gamma^\sigma_{\mu\tau}\Gamma^\tau_{\nu\rho}\right) \tag{1.3}$$

Such gravitational Chern-Simons term leads to a new massive, propagating degree of freedom. However, 3D TMG in AdS_3 is not well-defined for generic value μl, either because of the instability or negative energy for black hole.

In 2008, W. Li, W. Song and A. Strominger found that at the critical point $\mu l = \pm 1$, 3D TMG in AdS_3 could be well-defined. In this case, both local massive mode and the left-moving graviton are just pure gauge, and The only physical degree of freedom is the right-moving boundary graviton such that the gravity becomes chiral. It was further conjectured that the chiral gravity is holographically dual to a 2D chiral CFT after imposing self-consistent Brown-Henneaux boundary condition.

It would be interesting to investigate how the HS field coupled to topologically massive gravity and study the nature of the theory at the critical point.

2 High spin TMG

To study the high spin TMG with arbitrary spin n, we start from the following action

$$S_{TMG} = (1 - \frac{1}{\mu l})S_{CS}[A] - (1 + \frac{1}{\mu l})S_{CS}[\bar{A}] - \frac{k}{4\pi\mu}\int tr(\beta \wedge (F - \bar{F})). \tag{2.1}$$

where we have the gauge curvature

$$F = dA + A \wedge A, \quad \bar{F} = d\bar{A} + \bar{A} \wedge \bar{A}. \tag{2.2}$$

and one-form Lagrangian multiplier β. The gauge field A, $\bar{A}$ and the Lagrangian multiplier β are in the adjoint representation of the corresponding group, which is chosen to be $SL(n,R) \times SL(n,R)$ to describe the high spin fields from spin 2 to n. The last term is a Lagrangian multiplier. The imposed condition $F = \bar{F}$ looks strange, but it is nothing but torsion-free condition. The action describes all the high spin fields coupled to topological massive gravity: there are not only topologically Chern-Simons term for graviton, but also the similar parity-breaking Chern-Simons terms for higher spin fields.

In AdS_3 background, the equation of fluctuations are

$$\begin{aligned}
G(da + a \wedge A + A \wedge a) &= d\beta + \beta \wedge A + A \wedge \beta \\
\bar{G}(d\bar{a} + \bar{a} \wedge \bar{A} + \bar{A} \wedge \bar{a}) &= d\beta + \beta \wedge \bar{A} + \bar{A} \wedge \beta \\
da + a \wedge A + A \wedge a &= d\bar{a} + \bar{a} \wedge \bar{A} + \bar{A} \wedge \bar{a}
\end{aligned}$$

where we have defined $G = 2(\mu - 1), \bar{G} = 2(\mu + 1)$. This linearized equations describe free fluctuations of spin 2 to spin n. When $\beta = 0$, we can derive the Fronsdal equation

$$\begin{aligned}
\mathcal{F}_{\nu_1 \cdots \nu_s} \equiv \Box \Phi_{\nu_1 \cdots \nu_s} - \nabla_{(\nu_1|} \nabla^{\sigma} \Phi_{\sigma|\nu_2 \cdots \nu_s)} + \frac{1}{2} \nabla_{(\nu_1} \nabla_{\nu_2} \Phi'_{\nu_3 \cdots \nu_s)} \\
-(s^2 - 3s) \Phi_{\nu_1 \cdots \nu_s} - 2 g_{(\nu_1 \nu_2} \Phi'_{\nu_3 \cdots \nu_s)} = 0,
\end{aligned}$$

which describe the free massless HS fields in AdS_3 vacuum.

When $\beta \neq 0$, in principle we will get a third order differential equation for each spin s field. The calculation is quite tedious. For arbitrary spin $s \geq 2$, we finally obtain the equations of the physical fields

$$\mathcal{F}_{a_1 \cdots a_s} + \frac{1}{\mu s(s-1)} \epsilon_{(a_1|}^{\ \ bc} \nabla_b \mathcal{F}_{c|a_2 \cdots a_s)} = 0. \tag{2.3}$$

For every spin ≥ 3, the fluctuations could be decomposed into traceless and trace parts, each satisfying a third order differential equation. We can read massless left-moving, right-moving modes and a massive mode, for generic value of μl. At the critical point $\mu l = 1$, the massive modes are degenerate with the left-moving modes, both of which become pure gauge, and the only physical degrees of freedom are massless right-moving boundary modes. It seems that the theory is still chiral at the critical point. However there actually exist logarithmic modes at the critical point, which carry negative energy. The presence of the logarithmic modes breaks the chiral nature of the theory. It is not clear if we could find consistent boundary conditions to truncate these modes and make the theory chiral.

3 Open issues

One interesting issue is the gauge symmetry at non-linearized level. In our discussion, we showed that the gauge symmetry is recovered at the linearized level. At the non-linearized level, the gauge symmetry is broken, though it is still preserved on-shell. It is an interesting to investigate if the gauge symmetry is deformed in this case.

There are other interesting questions to ask. It is possible to generalize our construction to other gauge group. It would be interesting to study the asymptotic symmetry of our theory. It could be possible to exist other black hole solution in HS TMG theory. Moreover, our construction seems allow warped spacetime solution and open the window to study the HS field theory in warped spacetime.