Topics in Generalized Geometry

generalized contact structures and generalized Sasakian structures

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We provided a new approach of generalized Geometry on odd dimensional manifolds. Generalized complex structures are geometric structures which interpolate between complex structures and symplectic structures. Complex and symplectic structures arise as special ones of generalized complex structures. Generalized Kähler structures are the extended notion of ordinary Kähler structures which are equivalent to bihermitian structures and it also turns out that generalized Kähler structures are closely related to $N = (2, 2)$ supersymmetric sigma model in mathematical physics. Both generalized complex and generalized Kähler structures are geometric structures on an even dimensional manifold. In the talk, we discussed following three points of generalized geometry on an odd dimensional manifold M.

1. Generalized contact structures and generalized Sasakian structures are introduced from the view point of nondegenerate, pure spinors and the Clifford algebra on an odd dimensional manifolds M. Our approach is based on the one to one correspondent between generalized geometry on M and weighted generalized geometry of the cone $C(M)$ of M. Generalized contact structures corresponds to weighted Calabi-Yau structures on the cone which give weighted generalized complex structures. Generalized Sasakian structures also correspond to weighted generalized Kähler structures on the cone.

2. The stability theorem of generalized Kähler manifolds shows that generalized Kähler structures are stable under small deformations of generalized complex structures which is a generalization of the stability theorem of ordinary Kähler manifolds due to Kodaira-Spencer [2]. We discussed that the stability theorem of generalized Sasakian structures on M and showed that the stability theorem does hold under the certain cohomological condition. There are many ordinary Sasakian manifolds satisfying the cohomological con-

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dition. Then a weighted holomorphic Poisson structure β on the cone $C(M)$ gives rise to nontrivial deformations of generalized Sasakian structures starting from the ordinary Sasakian manifold M.

3. We obtained that the deformation complex of generalized contact structures is an elliptic differential complex and then the finite dimensional deformation space is constructed.

References

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